B.Sc. Semester-VI Examination, 2022-23

MATHEMATICS [Programme]

Course ID : 62118 **Course Code : SP/MTH/601/DSE-1B**

Course Title : Probability and Statistics

Time : 2 Hours Full Marks : 40

The figures in the right-hand margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

Notations and symbols have their usual meaning.

1. Answer any **five** of the following questions: 2×5=10

a) The random variable X is normal (50, 20). Find $P(|X-50|\le 20)$, given that $\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{1} e^{\frac{-x^2}{2}}dx = 0.8413$.

b) If $f(x)=ke^{-x}$, $(0\le x<-\infty)$ be the probability density function of a continuous random variable X, find the constant k.

c) If A and B are independent events, then prove that $\overline{A}$ and $\overline{B}$ are also independent.

d) Prove that variance of a random variable X can be put in the form $V(X)=E(X^2)-\{E(X)\}^2$.

e) Prove that $P(a<X\le b)=F(b)-F(a)$.

f) Write the probability density function of a normal variate $N(m,\sigma^2)$ and for the standardized variate.

g) If $E(X)$ exists for a random variable X, then prove for any linear function $a+bX$, a, b are finite, $E(a+bX)=a+bE(X)$.

h) Define null hypothesis.

2. Answer any **four** from the following questions: 5×4=20

a) For a binomial (6, p) variate

$$P(X=2)=9P(X=4),$$

find the value of p. 5

b) i) Two cards are drawn from a well-shuffled pack of cards. Find the probability that at least one of them is spade.

ii) The standard deviation of a Poisson distribution variate is 2. Calculate $P(X=3)$, use $e^{-4}=0.0183$. 3+2

c) If X follows an exponential distribution with parameter θ having the probability distribution function given by $f(x;\theta)=\frac{1}{\theta}e^{\frac{-x}{\theta}}$, $x>0$, $\theta>0$, find the maximum likelihood estimator of θ.

d) A random variable X has probability density function $f(x)=2x^2(1-x)$, $0<x<1$. Compute $P(|X-m|\geq 2\sigma)$ and compare it with the limit given by Tchebycheff's inequality.

e) Two non-negative random variable X and Y have joint density function $f(x, y)=\frac{1}{2}x^3e^{-x(y+1)}$, $x>0$, $y>0$. Find the correlation coefficient.

f) For random variable X and Y with the same mean, the two regression equations are $Y=aX+b$ and $X=\alpha Y+\beta$. Show that $\frac{b}{\beta}=\frac{1-a}{1-\beta}$.

3. Answer any **one** of the following questions:

10×1=10

a) i) If X and Y be correlated and U and V be defined by $U=X\cos\alpha+Y\sin\alpha$, $V=Y\cos\alpha-X\sin\alpha$, then prove that U and V will be uncorrelated if $\tan 2\alpha=\frac{2\rho\sigma_x\sigma_y}{\sigma_x^2-\sigma_y^2}$.

Also show that $\sigma_U\sigma_V=\sigma_X\sigma_Y\sqrt{1-\rho^2}$.

ii) Show that the mean deviation about the mean of a normal distribution (m, σ) is $\frac{\sqrt{2}}{\pi}\sigma$.

5+5=10

b) i) Verify whether the function

$$f(x)=\begin{cases}\frac{2}{9}x, & 0<x\leq 3\\ 0, & elsewhere\end{cases}$$

is a density function.

ii) Prove that the sample variance is a biased estimator of the population variance.

iii) Consider the random experiment of tossing a fair coin till a head appears for the first time. Let X be the number of tosses required. Obtain the distribution of X.

3+3+4

B.Sc. Semester-VI Examination, 2022-23

MATHEMATICS [Programme]

Course ID : 62118 **Course Code : SP/MTH/601/DSE-1B**

Course Title : Mechanics

Time : 2 Hours Full Marks : 40

The figures in the right-hand margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

Notations and symbols have their usual meaning.

1. Answer any **five** of the following questions: 2×5=10

   a) State the necessary and sufficient conditions of equilibrium of a system of coplanar forces.

   b) A uniform cubical box of edge 'a' placed on the top of a fixed sphere, the center of the face of the cube being in contact with the highest point of the sphere. What is the least radius of the sphere for which the equilibrium will be stable?

   c) Define the centre of gravity of a body. If a system of n bodies having masses $m_1, m_2, ..., m_n$ and having abscissas of their individual C.G parallel to x-axis are $\bar{x}_1, \bar{x}_2, \bar{x}_3, ..., \bar{x}_n$ respectively then write the x-coordinate of the combined C.G.

   d) What is the principle of virtual work? Explain it mathematically.

   e) Find moment of inertia of a uniform rod of length $2a$ and mass M about an axis through an extremity and perpendicular to it.

   f) What do you mean by degrees of freedom? In a three dimensional space there is a system of particles consisting of n particles. What is the degrees of freedom of the system?

   g) Find the minimum velocity with which a particle is projected horizontally from the earth's surface, so that the particle will circulate around the earth.

   h) Write down the differential equation of the path in pedal form moving under Inverse Square Law explaining the symbols used.

2. Answer any **four** of the following questions: 5×4=20

   a) A solid hemisphere of weight W rests in limiting equilibrium with its curved surface on a rough inclined plane and its plane face is kept horizontal by a weight P attached to a point on the rim. Prove that the coefficient of friction is $\frac{P}{\sqrt{W(2P+W)}}$.

b) Find the centre of gravity of the area included between the curve $y^2(2a-x)=x^3$ and its asymptote.

c) The moments of a system of forces about the points (0, 0), (a, 0), (0, a) are aw, $2aw$, $3aw$ respectively. Find the components of their resultant parallel to the co-ordinate axes and the equation to its line of action.

d) Find the time of oscillation of a compound pendulum consisting of a rod of mass m and length a, carrying at one end a sphere of mass m_1 and diameter $2b$, the other end of the rod being fixed.

e) State and prove the D'Alembert's Principle.

f) A particle falls under gravity in a medium in which the resistance is proportional to velocity. If the particle falls vertically downwards from a position of rest, then find the velocity and displacement in time t.

3. Answer any **one** of the following questions:

$10\times1=10$

a) i) A particle is projected at angle α to the horizontal direction in a resisting medium in which the resistance is proportional to the velocity at the point. Obtain the equation of the path of the particle.

ii) Discuss when a body will be called stable and unstable equilibrium. 8+2

b) i) A system of coplanar forces has the total moments H, $2H$ respectively about points whose co-ordinates are ($2a$, 0), (0, a) referred to fixed rectangular axes. The total resolved parts of the forces along the line $y=x$ vanishes. Find the points in which the line of action of the resultant meets the co-ordinate axes.

ii) Forces P, Q, R act along three non-intersecting edges of a cube. Find the central axis. 5+5

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